1.3 One-to-One Functions & Inverse Functions

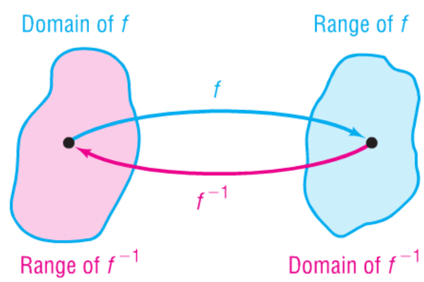
One-to-one Function*:** A function is ***one-to-one if for any value of x there is exactly one y (otherwise it wouldn't be a function), and for any value of y, there is exactly one x.

Example: Determine whether the following functions are one to one.

a) $\{(-2,6),(-1,3),(0,2),(1,5),(2,8)\}$

b) $\{(1,1),(2,4),(3,9),(0,0),(-1,1),(-2,4)\}$

Inverse Function*:** Two functions are ***inverses if and only if whenever one function contains the element (a,b), the other function contains the element (b,a). If f is a one-to-one function, the correspondence from the range of f back to the domain of f is called the ***inverse function*** of f. The inverse of f is abbreviated f^{-1}.



★ **Domain of f = Range of f^{-1}** **Range of f = Domain of f^{-1}**

Example: Find the inverse of the following one-to-one function: $\{(2,3),(4,5),(6,8),(9,10),(12,14)\}$

If we start with *x*, apply *f*, and then apply f^{-1}, we get *x* back again.
If we start with *x*, apply f^{-1}, and then apply *f*, we get *x* back again.
What f does, f^{-1} undoes, and vice versa. In other words,

$f^{-1}(f(x)) = x$, where *x* is in the domain of f.

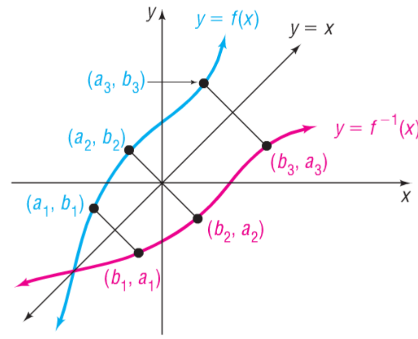
$f(f^{-1}(x)) = x$ where *x* is in the domain of f^{-1}.

To verify that two functions are inverses, show that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

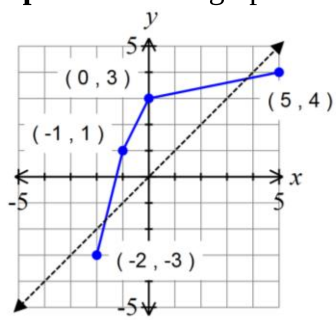
Example: Verify that the inverse of $f(x) = \dfrac{2}{x+5}$ is $f^{-1}(x) = \dfrac{2}{x} - 5$.

Example: Verify that the inverse of $f(x) = \sqrt[3]{2x}$ is $f^{-1}(x) = \dfrac{x^3}{2}$.

Theorem: The graph of a function f and the graph of its inverse f^{-1} are symmetric with respect to the line $y = x$.



Example: Draw the graph of the inverse function.



Finding the Inverse of a Function

1. Rewrite $f(x)$ as y in the original equation.
2. Interchange x and y.
3. Solve for y.
4. Replace y with the notation $f^{-1}(x)$.

Example: Find the inverse. State the domain and range of $f(x)$ and the domain and range of $f^{-1}\ x$.

a) $f(x) = -3x + 1$

b) $f(x) = \dfrac{2x+3}{5x-4}$